We may note that this expression is close to the effective and apparent stress values:

\[ S_a = 9 S_{eff} = \frac{\mu E_{strain}}{M_0}. \quad (10.24) \]

**REFERENCES**


**Chapter 11 Energy Budget of Earthquakes and Seismic Efficiency**

Hiroko Kanamori

11.1 INTRODUCTION

An earthquake is a rupture process occurring in Earth's interior under tectonic stresses caused primarily by plate motion. During an earthquake, the stress on a fault changes in a complex fashion, and the potential energy (strain energy and gravitational energy) stored in Earth is released as seismic waves. The ratio of the released seismic wave energy to the potential energy is the seismic efficiency which is determined by how the stress changes as a function of time during seismic rupture. A traditional method to investigate the stress variation on the fault plane is to determine the rupture pattern and slip function on a fault plane by inversion of observed seismograms. The stress can be inferred from the slip function. However, a fault plane is mechanically heterogeneous and the rupture pattern is very complex in both space and time. As a result, it is not possible to determine every detail of the rupture pattern. Another approach to this problem is to examine the energy budget of earthquakes and the efficiency. The amount of radiated energy increases with the slip velocity, which is proportional to the driving stress. Thus, by measuring the total radiated energy, we can obtain useful information about the state of stress during seismic faulting. In this approach, we do not determine the details of slip function at every point on the fault but, instead, determine the total energy radiated from the entire fault. This is somewhat similar to determining thermodynamic parameters (pressure, temperature, etc.) of a system without determining the motion of individual molecules in it. We will take the latter approach to investigate the physical processes associated with earthquakes.

11.2 ENERGY BUDGET OF EARTHQUAKES

We first consider the gross energy budget. We need to consider strain energy, \( E_s \), and gravitational energy, \( E_g \). The sum of \( E_s \) and \( E_g \) is the potential energy, \( W \). During an earthquake, three energies are involved: the wave...
energy $E_r$, frictional energy loss $E_f$, and fracture energy $E_c$. The wave energy is the energy radiated as seismic waves (body waves and surface waves) and can be measured with seismological methods. The frictional energy loss is the thermal energy caused by frictional stress on the fault plane during slippage. The fracture energy is the energy required to cause fracture near the end of a fault during rupture. We write $E_{rc}$ for the sum of $E_r$ and $E_c$, and call it nonradiated energy. Then the energy budget can be written as

$$\Delta (E_r + E_f) = E_{rc} + E_f + E_c \tag{11.1}$$

or

$$\Delta W = E_f + E_c. \tag{11.2}$$

In other words, an earthquake process transfers the potential energy to wave energy and nonradiated energy. Unfortunately, only $E_f$ can be directly measured with seismological methods. Others need to be either inferred or estimated from other data.

### 11.3 STRESS ON A FAULT PLANE

The stress on a fault plane drops from the initial (before an earthquake) stress, $\sigma$, to the final (after the earthquake) stress, $\sigma_f$, on the fault plane. During slippage, the stress is equal to the (dynamic) frictional stress, $\sigma_d$. In the simplest case, $\sigma_d$ is constant, but in general, $\sigma_d$ varies during faulting. Here, we define $\sigma_d$ as the average of the frictional stress during faulting. The final stress can be either smaller or larger than the frictional stress depending on how fault motion stops (Hossein, 1955; Brune, 1970).

The change in the potential energy $\Delta W$ can be written as (e.g., Knopoff, 1958; Kostrov, 1974; Dahlen, 1977; and Savage and Walsh, 1978)

$$\Delta W = DS(\sigma + \sigma_f)/2 \tag{11.3}$$

where $D$ and $S$ are the average slip (offset) and the fault area, respectively. The frictional energy loss is given by

$$E_f = \sigma_d DS. \tag{11.4}$$

These relations can be understood easily using a simple spring system as an analog of seismic faulting. The three stresses $\sigma_r$, $\sigma_d$, and $\sigma_f$ cannot be determined individually with seismological methods, but the differences,

$$\Delta \sigma = \sigma_0 - \sigma_1 \tag{11.5}$$

and

$$\Delta \sigma_f = \sigma_0 - \sigma_f \tag{11.6}$$

can be determined with seismological methods. $\Delta \sigma$ is the difference in stress on the fault plane before and after an earthquake and is called the static stress drop. $\Delta \sigma_f$ is called the dynamic stress drop, which is the stress that drives fault motion. The static stress drop, $\Delta \sigma_f$, can be estimated from $D$ and fault dimension, which can be inferred from $S$ and can be written as

$$\Delta \sigma = C_r \mu D/S^{1/2} \tag{11.7}$$

where $C_r$ is a constant of the order of unity determined by the geometry of the fault (Chinnery, 1964).

If $\sigma_d$ is constant, the dynamic stress drop, $\Delta \sigma_d$, is proportional to the particle velocity of fault motion $\dot{u} = D/2$ and can be written as

$$\Delta \sigma_d = C_r \mu D/\beta, \tag{11.8}$$

where $C_r$ is a constant that depends on the dynamic model used, but is in general of the order of unity (Brune, 1970). If $\sigma_d$ is not constant, Eq. (11.8) holds only approximately. The determination of $\Delta \sigma_d$ is more difficult than that of $\Delta \sigma$. From the observations of particle motion velocity, it is generally considered that $\Delta \sigma_d$ is about 10 to 100 bar. As we will show later, the dynamic stress drop can also be estimated from the energy measured from radiated waves.

### 11.4 SEISMIC MOMENT AND RADIATED ENERGY

An important seismological parameter is the seismic moment $M_0$, which is defined by

$$M_0 = \mu DS \tag{11.9}$$

and can be estimated primarily from the observed amplitude of seismic waves (Aki, 1966).

From (11.7) and (11.9), we obtain

$$M_0 = C_r \mu D \sigma_f S^{1/2}. \tag{11.10}$$

Figure 11.1 shows the relation between $M_0$ and $\sigma_f$, from which we can conclude that $\Delta \sigma_f$, is, for most large events, 10 to 100 bar.

The radiated energy can be estimated by integrating the energy flux,

$$E_r = \int \rho \dot{u}^2 dS \tag{11.11}$$

where $\dot{u}$ is the particle velocity of seismic wave, and $\rho$ and $\beta$ are density and $S$-wave velocity, respectively (Gutenberg, 1956; Bilbry, 1966; Haskell, 1964).
energy \( E_F \), frictional energy loss \( E_x \), and fracture energy \( E_G \). The wave energy is the energy radiated as seismic waves (body waves and surface waves) and can be measured with seismological methods. The frictional energy loss is the thermal energy caused by frictional stress on the fault plane during slippage. The fracture energy is the energy required to cause fracture near the end of a fault during rupture. We write \( E_F \), for the sum of \( E_x \) and \( E_G \), and call it nonradiated energy. Then the energy budget can be written as

\[
\Delta (E_F + E_x) = E_x + E_G
\]

or

\[
\Delta W = E_x + E_G.
\]

In other words, an earthquake process transfers the potential energy to wave energy and nonradiated energy. Unfortunately, only \( E_x \) can be directly measured with seismological methods. Others need to be either inferred or estimated from other data.

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The stress on a fault plane drops from the initial (before an earthquake) stress, \( \sigma_i \), to the final (after the earthquake) stress, \( \sigma_f \), on the fault plane. During slippage, the stress is equal to the (dynamic) frictional stress, \( \sigma_l \). In the simplest case, \( \sigma_l \) is constant, but in general, \( \sigma_l \) varies during faulting. Here, we define \( \sigma_l \) as the average of the frictional stress during faulting. The final stress can be either smaller or larger than the frictional stress depending on how fault motion stops (Housen, 1955; Brune, 1970).

The change in the potential energy \( \Delta W \) can be written as (e.g., Koons, 1958; Kostrov, 1974; Dahlen, 1977; and Savage and Walsh, 1978)

\[
\Delta W = D(S(\sigma_l + \sigma_l)/2)
\]

where \( D \) and \( S \) are the average slip (offset) and the fault area, respectively. The frictional energy loss is given by

\[
E_x = \sigma_l DS.
\]

These relations can be understood easily using a simple spring system as an analog of seismological faulting. The three stresses \( \sigma_m \), \( \sigma_l \), and \( \sigma_f \) cannot be determined individually with seismological methods, but the differences,

\[
\Delta \sigma_l = \sigma_l - \sigma_i
\]

and

\[
\Delta \sigma_f = \sigma_f - \sigma_l
\]

can be determined with seismological methods. \( \Delta \sigma_l \) is the difference in stress on the fault plane before and after an earthquake and is called the static stress drop. \( \Delta \sigma_f \) is called the dynamic stress drop, which is the stress that drives fault motion. The static stress drop, \( \Delta \sigma_l \), can be estimated from \( D \) and fault dimension, which can be inferred from \( S \) can be written as

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\Delta \sigma_l = C_l \mu D/S^{1/2}
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where \( C_l \) is a constant of the order of unity determined by the geometry of the fault (Chinnery, 1964).

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\[
\Delta \sigma_f = C_f \mu D/\beta
\]

(11.8)

where \( C_f \) is a constant that depends on the dynamic model used, but is in general of the order of unity (Brune, 1970). If \( \sigma_f \) is not constant, Eq. (11.8) holds only approximately. The determination of \( \Delta \sigma_f \) is more difficult than that of \( \Delta \sigma_l \). From the observations of particle motion velocity, it is generally considered that \( \Delta \sigma_f \) is about 10 to 100 bar. As we will show later, the dynamic stress drop can also be estimated from the energy measured from radiated waves.

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\]

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Figure 11.1 shows the relation between \( M_o \) and \( S \), from which we can conclude that \( \Delta \sigma_f \) is, for most large events, 10 to 100 bar. The radiated energy can be estimated by integrating the energy flux,

\[
E_r = \int \rho \beta |\dot{u}|^2 dS,
\]

(11.11)

where \( \dot{u} \) is the particle velocity of seismic wave, and \( \rho \) and \( \beta \) are density and S-wave velocity, respectively (Gutenberg, 1956; Bath, 1966; Haskell, 1964).
The surface integral is taken over a surface surrounding the source. Since most of the seismic wave energy is carried by S waves, the energies carried by other types of waves are ignored here. Although Eq. (11.11) appears simple and straightforward, the evaluation of this integral in practice is difficult because of the complex wave propagation effects in Earth.

### 11.5 SEISMIC EFFICIENCY AND RADIATION EFFICIENCY

Seismic efficiency \( \eta \) is defined by

\[
\eta = \frac{E_s}{\Delta W} = \frac{(\Delta W - E_f - E_o)}{\Delta W}.
\]  

(11.12)

Because \( \Delta W \), \( E_f \), and \( E_o \) cannot be determined with seismological methods, we cannot determine \( \eta \) directly from seismological data.

From (11.3) and (11.5) we can write

\[
\Delta W = DS(\sigma_e + \sigma_s)/2 = DS(\sigma_e - \sigma_s)/2 + DS\sigma_e = DS\Delta \sigma_e/2 + DS\sigma_s.
\]  

(11.13)

The first term of the right-hand side of (11.13),

\[
\Delta W_1 \approx \Delta \sigma_e DS/2 = \Delta \sigma_e M_o/2\mu
\]  

(11.14)
The surface integral is taken over a surface surrounding the source. Since most of the seismic wave energy is carried by S waves, the energies carried by other types of waves are ignored here. Although Eq. (11.11) appears simple and straightforward, the evaluation of this integral in practice is difficult because of the complex wave propagation effects in Earth.

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Because \( \Delta W \), \( E_s \), and \( E_o \) cannot be determined with seismological methods, we cannot determine \( \eta \) directly from seismological data.

From (11.3) and (11.5) we can write

\[
\Delta W = DS(\sigma_e + \sigma_t)/2 = DS(\sigma_e - \sigma_t)/2 + DS\sigma_t = DS\Delta \sigma / 2 + DS\sigma_t.
\]

The first term of the right-hand side of (11.13),

\[
\Delta W_1 = \Delta \sigma DS / 2 = \Delta \sigma M_o / 2\mu
\]

can be estimated from the static stress drop, \( \Delta \sigma \). Thus, the radiation efficiency \( \eta_r \) (Husacine and Randall, 1970) defined by

\[
\eta_r = \frac{E_r}{\Delta W_1}
\]

can be estimated.

Since \( \sigma_t \geq 0 \) for any reasonable earthquake model (i.e., no significant overshoot), \( \Delta W \geq \Delta W_1 \); hence,

\[
\eta_r \geq \eta.
\]

Thus, the radiation efficiency, \( \eta_r \), that can be estimated from \( \Delta W_1 \) and \( E_r \) (Eq. (11.15)) can also be called the maximum seismic efficiency.

### 11.6 RELATION BETWEEN EFFICIENCY AND RUPTURE SPEED

The radiation efficiency is related to the rupture speed \( V \) (Husacine and Randall, 1976). For simplicity, we use a Mode III (longitudinal shear) crack model in the following, but we can develop a similar argument for other crack geometries.

First, we consider a crack with a width \( 2c \) under uniform stress \( \sigma_0 \) and assume that there is no friction and the stress drops to 0.

Using the elastostatic theory for a Mode III crack, we obtain the total strain energy change per unit length of crack due to crack formation as

\[
\Delta W = \sigma_0 DS / 2 = \pi c^2 \sigma_0^2 / 2\mu.
\]

The static energy release rate \( G^* \) is given by

\[
G^* = K^2 / 2\mu = \pi c \sigma_0^2 / 2\mu,
\]

where \( \mu \) is the rigidity and \( K = \pi c (\sigma_0)^{1/3} \) is the stress intensity factor (Dmowska and Rice, 1986; Lawn, 1993; Freund, 1998). Since \( G^* \) is the energy required to produce a unit area of the crack at one of the crack tips, \( \Delta W_1 \) and \( G^* \) are related by

\[
d(\Delta W) \approx 2G^*dc
\]

Following Kostrov (1966), Edelshieb (1969), and Freund (1972), the energy release rate, \( G \), for a crack extending at a rupture speed speed \( V \) is given approximately by

\[
G = G^* g(V),
\]
where $g(V)$ is a universal function of $V$. For a Mode III crack, it is given by
\[
g(V) = \left(\frac{V}{V_0 + V_1}\right)^{3/2}
\]
(11.21)

where $\beta$ is S-wave velocity.

Since there is no friction in this case, the efficiency $\eta$ is given by
\[
\eta = \frac{(\Delta W - E_0)}{\Delta W}
\]
(11.22)

where the fracture energy $E_0$ is given by
\[
E_0 = \int G dS = 2 \int G' g(V) dc = g(V) \int d(\Delta W) - g(V) \Delta W.
\]
(11.23)

Here, the rupture speed is assumed constant. Thus, from (11.22) and (11.23),
\[
\eta = 1 - g(V)
\]
(11.24)

In the preceding discussion, there is no friction on the fault plane, and the radiation efficiency and the seismic efficiency are identical.

Next, we include friction $\sigma_f$ on the fault plane. In actual faulting, $\sigma_f$ is likely to vary during faulting, but we need to assume it to be constant in our simple model. As a result, the final stress $\tau_f$ is equal to $\sigma_f$. Then,
\[
\Delta W = (\sigma_{0} + \sigma_f) DS/2 = (\sigma_0 - \sigma_f) DS/2 + \sigma_f DS
\]
(11.25)

and
\[
G^* = \pi c (\sigma_0 - \sigma_f)^2 / 2 \mu
\]
(11.26)

\[
\Delta W_l = \Delta W - E_f = \Delta W - \sigma_f DS = (\sigma_0 - \sigma_f) DS/2 = \pi c (\sigma_0 - \sigma_f)^2 / 2 \mu
\]
(11.27)

From (11.26) and (11.27),
\[
d(\Delta W_l) = 2 G^* dc
\]
(11.28)

and the fracture energy is
\[
E_0 = \int G dS = 2 \int G' g(V) dc = g(V) \int d(\Delta W_l) - g(V) \Delta W_l.
\]
(11.29)

Then the radiation efficiency and the seismic efficiency are given by
\[
\eta_R = \frac{(\Delta W - E_f - E_0)}{\Delta W_l} = \frac{(\Delta W_l - E_f)}{\Delta W_l} = 1 - g(V)
\]
(11.30)

and
\[
\eta_s = (\Delta W - E_f - E_0)/\Delta W_l = (\Delta W_l - E_f)/\Delta W_l = \frac{1}{1 + E_f/\Delta W_l} / (1 - g(V)) \leq \eta_s.
\]
(11.31)

Equations (11.21) and (11.30) mean that if rupture speed is close to S-wave velocity, $g(V)$ approaches 0, and $E_0$ can be ignored compared with $\Delta W_l$.

### 11.7 EFFICIENCY OF SHALLOW EARTHQUAKES

It is generally accepted that the average rupture speed is 70 to 85% of the local S-wave velocity. This suggests that, for shallow earthquakes, the fracture energy $E_0$ can be ignored.

If $E_0$ is ignored, we obtain from (11.2) and (11.4)
\[
E_R = \Delta W - E_f = \left[\left(\sigma_0 + \sigma_f\right)/2 - \sigma_f\right] DS
\]
\[
= \left[\left(\sigma_0 - \sigma_f\right) - \left(\sigma_0 - \sigma_f\right)/2\right] DS
\]
\[
= \left(0.5 \sigma_0 - 0.5 \sigma_f\right) DS.
\]
(11.32)

Using (11.9), this can be written as
\[
E_R/M_0 = (2 \Delta \sigma_l - \Delta \sigma_s)/2 \mu.
\]
(11.33)

Since both $M_0$ and $E_R$ can be directly determined from seismic observations, this relationship can be used to obtain a useful constraint on stresses involved in earthquakes. However, as mentioned earlier, accurate determination of the radiated energy, $E_R$, is difficult and large errors are involved. Figure 11.2 shows some examples. Especially notable is the systematic difference of more than 20% between the results obtained from local and regional data (shown in Fig. 11.2a) and teleseismic data (shown in Fig. 11.2b). At present, this problem has not been resolved. The modern high-density regional network provides a data set for reliable determination of the $E_R/M_0$ ratio. Figure 11.3 shows the result of a recent study in southern California using close-in data obtained from a high-density broadband seismic network, TriNet, and the method described in Kanamori et al. (1993). Because of the limited bandwidth of the instrument and the loss of high-frequency energy during wave propagation, the energy estimates for earthquakes with $M_0 < 4.5$ (i.e., $M_0 < 7 \times 10^{13}$ Nm) are not reliable and are not considered here. The magnitude, $M_0$, is related to $M_b$ by the relation (Kanamori, 1977)
\[
M_b = \log M_0/(4.5) - 10.7 \quad (M_0 \text{ in dyn-cm}).
\]
(11.34)
where \( g(V) \) is a universal function of \( V \). For a Mode III crack, it is given by
\[
g(V) = \left( \frac{\beta - V}{\beta + V} \right)^{3/2} \tag{11.21}
\]
where \( \beta \) is S-wave velocity.

Since there is no friction in this case, the efficiency \( \eta \) is given by
\[
\eta = \frac{\Delta W - E_c}{\Delta W} \tag{11.22}
\]
where the fracture energy \( E_c \) is given by
\[
E_c = \int G dS = 2 \int_0^r G^* g(V) dc = g(V) \int d(\Delta W) - g(V) \Delta W. \tag{11.23}
\]

Here, the rupture speed is assumed constant.

Thus, from (11.22) and (11.23),
\[
\eta = 1 - g(V) \tag{11.24}
\]

In the preceding discussion, there is no friction on the fault plane, and the radiation efficiency and the seismic efficiency are identical.

Next, we include friction \( \sigma_r \) on the fault plane. In actual faulting, \( \sigma_r \) is likely to vary during faulting, but we need to assume it to be constant in our simple model. As a result, the final stress \( \sigma_f \) is equal to \( \sigma_r \). Then,
\[
\Delta W = (\sigma_0 + \sigma_r) DS/2 = (\sigma_0 - \sigma_r) DS/2 + \sigma_r DS \tag{11.25}
\]
and
\[
G^* = \pi c (\sigma_0 - \sigma_r)^2 / 2 \mu \tag{11.26}
\]

and
\[
\Delta W_i = \Delta W - E_f = \Delta W - \sigma_r DS = (\sigma_0 - \sigma_r) DS/2 = \pi c (\sigma_0 - \sigma_r)^2 / 2 \mu \tag{11.27}
\]

From (11.26) and (11.27),
\[
d(\Delta W_i) = 2G^* dc \tag{11.28}
\]

and the fracture energy is
\[
E_c = \int G dS = 2 \int_0^r G^* g(V) dc = g(V) \int d(\Delta W_i) = g(V) \Delta W_i. \tag{11.29}
\]

Then the radiation efficiency and the seismic efficiency are given by
\[
\eta_r = \frac{\Delta W - E_f - E_c}{\Delta W} = \frac{\Delta W_i - E_c}{\Delta W_i} = 1 - g(V) \tag{11.30}
\]

and
\[
\eta = \frac{\Delta W - E_f - E_c}{\Delta W} = \frac{\Delta W_i / \Delta W}{(\Delta W_i - E_c) / \Delta W_i} \tag{11.31}
\]

Equations (11.21) and (11.30) mean that if rupture speed is close to S-wave velocity, \( g(V) \) approaches 0, and \( E_c \) can be ignored compared with \( \Delta W_i \).

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If \( E_c \) is ignored, we obtain from (11.2) and (11.4)
\[
E_R = \Delta W - E_f = \left( \sigma_0 + \sigma_r \right) DS/2 - \sigma_r DS \tag{11.32}
\]

\[
= (\sigma_0 - \sigma_r - \sigma_f) DS / 2 \tag{11.33}
\]

Using (11.9), this can be written as
\[
E_R / M_c = (2 \Delta \sigma_f - \Delta \sigma_i) / 2 \mu. \tag{11.33}
\]

Since both \( M_c \) and \( E_R \) can be directly determined from seismic observations, this relationship can be used to obtain a useful constraint on stresses involved in earthquakes. However, as mentioned earlier, accurate determination of the radiated energy, \( E_R \), is difficult and large errors are involved. Figure 11.2 shows some examples. Especially notable is the systematic difference of more than a factor of 10 between the results obtained from local and regional data (shown in Fig. 11.2a) and teleseismic data (shown in Fig. 11.2b). At present, this problem has not been resolved. The modern high-density regional network provides a data set for reliable determination of the \( E_R / M_c \) ratio. Figure 11.3 shows the result of a recent study in southern California using close-in data obtained from a high-density broadband seismic network, TriNet, and the method described in Kanamori et al. (1993). Because of the limited passband of the instrument and the loss of high-frequency energy during wave propagation, the energy estimates for earthquakes with \( M_c < 4.5 \) (i.e., \( M_c < 7 \times 10^{15} \) Nm) are not reliable and are not considered here. The magnitude, \( M_c \), is related to \( M_b \) by the relation (Kanamori, 1977)
\[
M_c = \log M_b / 1.5 - 10.7 \quad (M_b \text{ in dyn-cm}). \tag{11.34}
\]
As shown in Fig 11.3, the ratio does not depend on $M_a$ (or $M_e$) and is approximately constant at about $5 \times 10^{-3}$ to $2 \times 10^{-4}$ for $M_e \geq 4.5$ earthquakes. From Eq. (11.33), we can write

$$\Delta \sigma_f = \Delta \sigma_c / 2 + c_1$$

(11.35)
Figure 11.2 The relation between radiated energy $E_R$ and the seismic moment $M_0$, (a) The result from local and regional data, and (b) the result obtained from teleseismic data (Singh and Ordaz, 1994).

As shown in Fig 11.3, the ratio does not depend on $M_0$ (or $M_s$) and is approximately constant at about $5 \times 10^{-3}$ to $2 \times 10^{-4}$ for $M_s$ $\geq$ 4.5 earthquakes. From Eq. (11.33), we can write

$$\Delta \sigma_r = \Delta \sigma / 2 + c_1$$  \hspace{1cm} (11.35)

where $c_1 = \mu (E_R/M_0)$. If $\mu = 3 \times 10^3$ bar is used for the rigidity of the crust, $c_1$ ranges from 15 to 60 bars. Then for a reasonable range of $\Delta \sigma$, 30 to 10 bar, $\Delta \sigma_r$ varies from 30 to 110 bar, and the ratio $\Delta \sigma_r / \Delta \sigma$ does not differ much from unity. Since the scatter of the $E_R/M_0$ ratio is large, this conclusion should not be taken rigorously. It should be interpreted as meaning that the dynamic and static stress drops are about the same order, 30 to 100 bar.

As mentioned earlier, the frictional stress, $\sigma_f$, does not appear in Eq. (11.33) and cannot be determined from seismic data, but it can be inferred from a simple shear frictional heating model. The thermal energy given by Eq. (11.4) is used to raise the temperature of a small volume, $w$, of rock surrounding the slip zone, where $w$ is the thickness of the slip zone. Then the temperature rise $\Delta T$ can be given by

$$\Delta T = \sigma_f D / C \rho \omega,$$  \hspace{1cm} (11.36)

where $C$ is the heat capacity and $\rho$ is the density. If we assume that melting or some other thermal processes (e.g., thermal pressurization) occurs at $\Delta T \sim 1000^\circ$C, then, combining Eqs. (11.7) and (11.36), the minimum frictional stress, $\sigma_f$, that causes melting is given by

$$\sigma_f \text{ (in bar)} = 2.8 \times 10^5 w/10^{3.5} M_s.$$  \hspace{1cm} (11.37)
where \( C = 1 \text{ J/g°C}, \Delta \sigma = 100 \text{ bar}, \rho = 2.6 \text{ g/cm}^3, \mu = 3 \times 10^5 \text{ bar}, \) and \( C_i = 7\pi^{5/2}/16 \) (the constant for a circular crack) are used, and \( w \) is measured in centimeters.

When melting occurs, the frictional property would change drastically. In the beginning of melting when solid material still exists in the melt, friction may rise temporarily, but in the advanced stage of melting, the friction would drop. Once melting occurs, the friction will drop, and no more heating occurs; then melting will cease and friction will increase again. Thus, macroscopically, the whole process could occur at an equilibrium state at a low stress level. Using Eq. (11.37), we find that, if \( w = 1 \text{ mm}, \eta_T \) becomes comparable to \( \Delta \sigma \) and \( \Delta \sigma \) for earthquakes with \( M_c \geq 5 \); that is, an earthquake is an almost complete stress release process at a stress level similar to \( \Delta \sigma \). A small \( w \), less than a few millimeters, is often suggested from field evidence, especially from well-preserved pseudotachylites, a glassy material found in fault zones (Scottson, 1975; Obata and Karato, 1995; Hull, 1988; Otsuki, 1998). For earthquakes with \( M_c < 4 \), melting is unlikely to occur, and the friction, \( \sigma_c \) can be high. It is interesting to note that the ratio \( E_p/M \) becomes very small for small earthquakes. Abercrombie (1995) showed that for earthquakes with \( M_c < 2 \), the \( E_p/M \) ratio is of the order of \( 10^{-5} \), almost two orders of magnitude smaller than that for large earthquakes (Fig. 11.4). Since the \( E_p/M \) ratio does not explicitly depend on \( \sigma_c \) [Eq. (11.33)], a large \( \sigma_c \) does not necessarily result in small \( E_p/M \). However, if the friction drops gradually, \( \Delta \sigma \) becomes relatively small compared with \( \Delta \sigma \), resulting in small \( E_p/M \). Also, if the fracture energy, \( E_f \), is not small, Eq. (11.33) should be written as

\[
\frac{E_p}{M} = \frac{\Delta \sigma \sigma_c}{\Delta \sigma_0} / 2 \mu - E_f/M_0 \tag{11.33}
\]

and the ratio \( E_p/M \) would decrease. For small earthquakes, the rupture speed has not been determined accurately so that the evidence for small \( E_p \) does not exist. Thus, the small \( E_p/M \) ratio for small earthquakes can be interpreted as due to gradual drop of friction, large fracture energy, or both (see Kanamori and Heaton, 2000, for more details on this).

Since the radiation efficiency is given from Eqs. (11.14) and (11.15) as

\[
\eta_T = \frac{2 \mu}{\sigma_c} \frac{E_p}{\Delta \sigma_0} M_0, \tag{11.38}
\]

the small \( E_p/M \) for small earthquakes means that \( \eta_T \) is small. From Eqs. (11.12), (11.13), and (11.15),

\[
\eta = \frac{\sigma_c}{\sigma_c + \sigma_f} \eta_T. \tag{11.39}
\]

Thus, the seismic efficiency can be even smaller if \( \sigma_c \) is large and of the same order as \( \sigma_f \). This means that seismic efficiency is very different between small and large earthquakes.

In the foregoing, we assumed that \( w \) is small and that melting or other thermal processes generally reduces friction. The validity of these assumptions, however, is not yet established, and further studies are required.

### 11.8 Deep-Focus Earthquakes

It is usually difficult to accurately determine the size of the fault plane, \( S \), for deep-focus earthquakes. Because of this difficulty, the analysis we used for shallow earthquakes cannot be used. However, for the 1994 Bolivian earthquake \( (M_l = 8.3) \), the largest deep-focus earthquake ever recorded, the source parameters could be determined well enough to investigate the energy budget (Kanamori et al., 1998).

Among the important parameters are total seismic moment, \( M_0 = 3 \times 10^{21} \) Nm, average static stress drop, \( \Delta \sigma = 1 \) kbar, radiated energy \( E_r = 5 \times 10^{18} \) J, and rupture speed \( V = 1 \) km/sec, which is only 20% of the \( S \)-wave velocity. From these results, using Eqs. (11.14) and (11.15), we obtain \( \Delta W \approx 1.25 \times 10^{18} \) J and \( \eta_T \approx 0.024 \). Thus, the source process of the Bolivian earthquake
where $C = 1 \text{ J/g°C}$, $\Delta \sigma = 100 \text{ bar}$, $\rho = 2.6 \text{ g/cm}^3$, $\mu = 3 \times 10^7 \text{ bar}$, and $C_i = 7 \pi \nu^2/86$ (the constant for a circular crack) are used, and $w$ is measured in centimeters.

When melting occurs, the frictional property would change drastically. In the beginning of melting when solid material still exists in the melt, friction may rise temporarily, but in the advanced stage of melting, the friction would drop. Once melting occurs, the friction will drop, and no more heating occurs; then melting will cease and friction will increase again. Thus, macroscopically, the whole process could occur at an equilibrium state at a low stress level. Using Eq. (11.37), we find that, if $w = 1 \text{ mm}$, $\eta_w$ becomes comparable to $\Delta \sigma$ and $\Delta \sigma_0$ for earthquakes with $M_c \geq 5$; that is, an earthquake is an almost complete stress release process at a stress level similar to $\Delta \sigma$. A small $w$, less than a few millimeters, is often suggested from field evidence, especially from well-preserved pseudotachylites, a glassy material found in fault zones (Sibson, 1975; Obata and Karato, 1995; Hull, 1988; Otsuki, 1998). For earthquakes with $M_c < 4$, melting is unlikely to occur, and the friction, $\sigma_f$, can be high. It is interesting to note that the ratio $E_g/M_k$ becomes very small for small earthquakes. Abercrombie (1995) showed that for earthquakes with $M_c < 2$, the $E_g/M_k$ ratio is of the order of $10^{-6}$, almost two orders of magnitude smaller than that for large earthquakes (Fig. 11.4). Since the $E_g/M_k$ ratio does not explicitly depend on $\sigma_f$ (Eq. (11.33)), a large $\sigma_f$ does not necessarily result in small $E_g/M_k$. However, if the friction drops gradually, $\Delta \sigma_0$ becomes relatively small compared with $\Delta \sigma$, resulting in small $E_g/M_k$. Also, if the fracture energy, $E_f$, is not small, Eq. (11.33) should be written as

$$E_g/M_k = (2\Delta \sigma_0 - \Delta \sigma)/(2\mu - E_f/M_k)$$

(11.33)

and the ratio $E_g/M_k$ would decrease. For small earthquakes, the rupture speed has not been determined accurately so that the evidence for small $E_f$ does not exist. Thus, the small $E_g/M_k$ ratio for small earthquakes can be interpreted as due to gradual drop of friction, large fracture energy, or both (see Kanamori and Heaton, 2000, for more details on this).

Since the radiation efficiency is given from Eqs. (11.14) and (11.15) as

$$\eta_r = \frac{2\mu \ E_g}{\Delta \sigma_0 \ M_k},$$

(11.38)

the small $E_g/M_k$ for small earthquakes means that $\eta_r$ is small. From Eqs. (11.12), (11.13), and (11.15),

$$\frac{\sigma_f - \sigma_t}{\sigma_t + \sigma_r} \eta_r.$$  

(11.39)

Thus, the seismic efficiency can be even smaller if $\sigma_f$ is large and of the same order as $\sigma_r$. This means that seismic efficiency is very different between small and large earthquakes.

In the foregoing, we assumed that $w$ is small and that melting or other thermal processes generally reduces friction. The validity of these assumptions, however, is not yet established, and further studies are required.

### 11.8 DEEP-FOCUS EARTHQUAKES

It is usually difficult to accurately determine the size of the fault plane, $S$, for deep-focus earthquakes. Because of this difficulty, the analysis we used for shallow earthquakes cannot be used. However, for the 1994 Bolivian earthquake ($M_s = 8.3$), the largest deep-focus earthquake ever recorded, the source parameters could be determined well enough to investigate the energy budget (Kanamori et al., 1998).

Among the important parameters are the total seismic moment, $M_o = 3 \times 10^{21}$ Nm, average static stress drop, $\Delta \sigma = 1$ kbar, radiated energy $E_R = 5 \times 10^{16}$ J, and rupture speed $V = 1$ km/sec, which is only 20% of the S-wave velocity. From these results, using Eqs. (11.14) and (11.15), we obtain $\Delta W_f = 1.25 \times 10^{21}$ J and $\eta_r = 0.024$. Thus, the source process of the Bolivian earthquake.
appears to be highly dissipative. Then, from Eqs. (11.2) and (11.3), the amount of nonradiated energy is

\[ E_N = \Delta W(1 - \eta_0) + \sigma_1 \Delta S. \] (11.40)

Since \( \eta \) is generally positive, the minimum nonradiated energy produced during the Bolivian rupture can be estimated as \( 1.2 \times 10^{14} \) J from the first term of (11.40), which is comparable to the thermal energy of the 1980 Mount St. Helens eruption. A simple calculation shows that this much energy is sufficient to melt a layer as thick as 30 cm. Thus, once rupture is initiated, melting can occur, which reduces friction and promotes fault slip.

The slow rupture velocity is also suggestive of a dissipative process. If \( V/\beta = 0.2 \), then \( 1 - \gamma(V') = 0.18 \), which suggests that the upper bound of seismic efficiency is 0.18 [Eq. (11.30)]. This is qualitatively consistent with the efficiency directly estimated from radiated energy.

At present, it is unclear whether the result obtained for the Bolivian earthquake is representative of deep-focus earthquakes or not. In the analysis just shown, the determinations of \( S \) and the rupture speed \( V' \) play a key role. Unfortunately, accurate determinations of these parameters for other deep-focus earthquakes are not available, however, the result suggests that melting appears to play a key role in the dynamics of large deep-focus earthquakes.

REFERENCES

Abercrombie, R. (1993). Earthquake source scaling relationships from \( 1 \) to \( 5 M_\text{w} \) using seismograms recorded at 2.5-km depth. J. Geophys. Res. 100, 24,015–24,036.


appears to be highly dissipative. Then, from Eqs. (11.2) and (11.3), the amount of nonradiated energy is

$$E_p = \Delta W (1 - \eta_p) + \sigma_p DS.$$  \hspace{1cm} (11.40)

Since $\eta_p$ is generally positive, the minimum nonradiated energy produced during the Bolivian rupture can be estimated as $1.2 \times 10^{46}$ J from the first term of (11.40), which is comparable to the thermal energy of the 1980 Mount St. Helens eruption. A simple calculation shows that this much energy is sufficient to melt a layer as thick as 30 cm. Thus, once rupture is initiated, melting can occur, which reduces friction and promotes fault slip.

The slow rupture velocity is also suggestive of a dissipative process. If $V/\beta = 0.2$, then $1 - \gamma(V') = 0.18$, which suggests that the upper bound of seismic efficiency is 0.18 [Eq. (11.3)]. This is qualitatively consistent with the efficiency directly estimated from radiated energy.

At present, it is unclear whether the result obtained for the Bolivian earthquake is representative of deep-focus earthquakes or not. In the analysis just shown, the determinations of $S$ and the rupture speed $V'$ play a key role. Unfortunately, accurate determinations of these parameters for other deep-focus earthquakes are not available, however, the result suggests that melting appears to play a key role in the dynamics of large deep-focus earthquakes.

REFERENCES


