134. Continuous Measurements of Gravity on Board a Moving Surface Ship

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A ship-borne vibration string gravity meter was constructed and put to the test in two voyages which were made in July and August, 1961, on board the “Takuyo” of the Hydrographic Office, Maritime Safety Agency, Japan. Throughout the whole streches of the voyages, the instrument worked perfectly well as it had been designed to and almost continuous records of satisfactorily accurate gravity data were obtained.

Prior to this, several ship-borne gravity meters which work on different principles had been built and tested by the authors, but none of them proved to be quite satisfactory for the purpose of regular observations. Much have been learned, however, through these experiments which were indispensable in designing the new instrument to be described in the present article.

The new gravity meter is of a vibration string type as shown schematically in Fig. 1 and is in its principle essentially similar to the one which was described by R. L. G. Gilbert. The weight $M$ is suspended from a thin metallic strip $L$ which under the tension $P$ due to the weight makes rapid chord vibrations. Unlike in the Gilbert’s instrument, the weight is connected also sideway to the supporting frame by means of two pairs of thin cross leaf springs $S$, so that no pendulous swinging of the weight can be caused by ship’s movements.

Any change in the value of gravity causes a corresponding change in the tension of the string and hence that in its vibration period. If the string were perfectly flexible and no elasticity were working...

Fig. 1. Pendulum

at the side cross springs, the change in period $T$ according to that in gravity is expressed by
\[ \Delta T/T = -\Delta g/2g. \]

Although much technical caution was paid in order to minimize these residual elastic contributions, they seem to be still present in no negligible amount and the exact way of dependence of $\Delta T$ on $\Delta g$ remains to be determined by experiments.

Both the string and the cross springs are of beryllium-copper. The string is a thin flat strip about $0.015\,\text{mm} \times 0.17\,\text{mm}$ in cross section and $30\,\text{mm}$ in length. The cross springs are $0.12\,\text{mm} \times 2\,\text{mm} \times 5\,\text{mm}$ in size. The weight is made of brass and has a mass of $25\,\text{gr}$. The string is placed in a narrow gap between two poles of a permanent magnet and its vibration is self-maintained by the coupling of the field with the weak intermittent electric current which is made to flow in the string according to its vibration through a transistor feedback circuit.

After this pendulum part is built, it is enclosed in a metallic case and put in an electric furnace to be heated up to $310^\circ\text{C}$ and annealed down from that temperature. This is done for the purpose of minimizing the creep of beryllium-copper and also of removing gases from the metallic parts of the pendulum support and the case. To the top of the case, a glass tube is cemented through which air and gases inside the case are pumped out. The glass tube is sealed when the inside pressure is reduced to about $10^{-4}\,\text{mm Hg}$.

The vibration of the string is approximately 1,800 cycles per second in frequency and extremely small in amplitude, being a few microns at its loop. The $Q$-value of the string vibration is as high as 25,000 and this is an essential thing for the vibration frequency to be assuredly stationary. At every time interval $\tau$ of 0.6 sec, or more precisely at every $1,024 = (2^{10})$ vibrations of the string, electric pulses are sent out and the time length between two such consecutive pulses is measured by means of a quartz crystal clock working with an approximate frequency of $30\,\text{kc per second}$. Since within the time interval $\tau$, the crystal makes approximately $30,000 \times 0.6 = 18,000$ vibrations, 1 vibration difference within $\tau$ corresponds roughly to a 100 mgal difference in gravity. The number of vibrations $n$ within each consecutive $\tau$ is automatically counted and is recorded in the binary system by perforating paper tape. The electric pulses sent out at the beginning and the end of $\tau$ are used to open and close the gate of the counters. The number $n$ represents the time needed for $1,024$ vibrations of the string as measured in terms of the vibration period of the crystal clock and in that sense may also be called the period $T$ of the string vibration.
In recording the number of vibrations, 16,384 = 2^{14} is taken off from \( n \) as being common to all, so that the number which is actually recorded on tape is \((n - 16,384)\). A series of 1,000 such continuous perforation records is obtained on tape in 10 minutes and this forms one set of observation data from which the gravity value is to be deduced.

The period of vertical acceleration \( \alpha \) to which the whole instrument is subjected on board a moving ship is about ten times as long as the time interval \( \tau \) of 0.6 sec, so that \( \alpha \) may well be regarded to be constant throughout it. The measured period \( T \) of the gravity meter string is therefore given by

\[
T = \frac{k}{\sqrt{g + \alpha}},
\]

where \( k \) is a constant. There are thus 1,000 values such as

\[
T_1 = \frac{k}{\sqrt{g + \alpha_1}},
\]

\[
T_2 = \frac{k}{\sqrt{g + \alpha_2}},
\]

\[
\cdots
dots
\]

\[
T_{1000} = \frac{k}{\sqrt{g + \alpha_{1000}}},
\]
in one set of observations. The assumption of constant \( g \) within 10 minutes is admissible considering the speed of the ship and the spatial gradient of gravity.

Since only the integral part of the vibration number of the crystal clock can be recorded digitally on tape, a device has to be developed in order to take care of its fractional parts which take place at the beginning and at the end of the time interval \( \tau \). For this purpose, two sets of transistor counter circuits are necessary which work at alternate intervals. The number counted by each counter is \((\text{the number of zero passage of the crystal vibration in the same direction within } \tau\text{ plus one})\), so that an accuracy of one count can be guaranteed of the total number of vibrations within any time interval. Since one count difference within \( \tau \) corresponds to a 100 mgal difference in gravity, the overall accuracy obtainable in 1,000 intervals or in 10 minutes is nominally 0.1 mgal. Experiments made in the laboratory where no disturbance is present showed this is actually so.

On board a ship, the string pendulum is mounted on a platform suspended in gimbals. The platform is stabilized by means of a gyroscope and a pair of torque motors which is controlled by two sets of mercury level accelerometers. The rotor of the gyroscope weighs about 1 kg and is driven by AC to make 12,000 revolutions per minute. The whole system is so constructed that the time derivative of the angle between the true vertical and the artificial vertical produced by this system is proportional to the horizontal
acceleration to which it is subjected at the same moment. There is therefore a phase difference of 90° between the angle and the horizontal acceleration and by this very reason, the undesirable effect of the oscillatory deviation of the gravity meter string from the true vertical on the measured gravity value is averaged out to be very small. All that remains to be required is to keep the artificial vertical with an accuracy of 5′ of arc and this can be realized without great difficulty.

The gimbal system is set up in a case which is covered by heat insulator styrofoam on all sides. The temperature inside the case is kept to be 42°±0.2°C by means of 4 sets of 48 pairs of thermoelements and a thermister which controls the system. The whole case is suspended and supported in a frame by means of rubber bands and dashpots from many directions. The frame itself is placed on the floor of a room or a cabin on a ship with shock-absorbers inbetween. Fig. 2 and 3 are photographs of the assembly.

The final important problem is how to find the period T with which the string would vibrate...
in case there were no ship's vertical motion from observed \( T_1, T_2, \ldots, T_{1000} \). In the relation
\[
T = k/\sqrt{g + \alpha},
\]
\( \alpha \) which is the momentary vertical acceleration of the ship sometimes reaches 100 gals. This means that the (signal)/(noise) ratio is as low as \( 10^{-3} \) if an accuracy of 1 mgal is aimed at. In order to smooth out the noise effect of the ship's accelerations, the overlapping averages were repeatedly taken of the 1,000 values of \( g + \alpha = k^2/T^2 \) as follows.

The first overlapping average taken is
\[
\bar{g}_t = \frac{1}{100} \sum_{j=-50}^{50} (g + \alpha_{t+j})
\]
and there are 900 such \( \bar{g}_t \) values. The second averaging which is applied to \( \bar{g}_t \) is
\[
\bar{g}_t = \frac{1}{150} \sum_{j=-75}^{75} \bar{g}_{t+j}
\]
and there are 750 such \( \bar{g}_t \) values. The third averaging which is applied to \( \bar{g}_t \) is
\[
\bar{g}_t = \frac{1}{200} \sum_{j=-100}^{100} \bar{g}_{t+j}
\]
and there are 550 such \( \bar{g}_t \) values. The value \( g^* \) which is adopted as final is
\[
g^* = \frac{1}{550} \sum_{j=-275}^{275} \bar{g}_t.
\]

The effect due to the ship's motion with an average period of 6 sec can be reduced to \( 1/150,000 \) of its original value by means of the averaging.

The above reasoning is true only when the functional relation
\[
T = k/\sqrt{g + \alpha}
\]
holds exactly. It has been found that this is not necessarily so and in order to obtain the real value of \( g \), a correction such as
\[
g = g^* \left\{ 1 - c \left( \frac{\alpha}{g} \right)^2 \right\}
\]
should further be applied to \( g^* \), where \( c \) is a constant. The value of \( c \) was experimentally found to be \( 0.25 \pm 0.03 \), although its physical nature is not quite clear and remains to be investigated.

The whole assembly of instruments was mounted on board the "Takuyo" of the Hydrographic Office, Maritime Safety Agency, in the master gyroscope room which is located very close to the center of the ship and slightly below the water line. The "Takuyo" is a 700 ton survey ship and is conducted by Captain S. Matsubara. From June 10th to 13th, the ship made a voyage from Tokyo down
south to near Miyakeshima and back and from July 31st to Aug. 17th, the other from Tokyo southwest to Kagoshima, Kyushu, and back. The distance covered was $90 \times 2$ nautical miles for the first voyage and $1,200 \times 2$ nautical miles for the second. During the voyages, the measurements of 10 minutes were taken with two minute pause inbetween. This means 5 observations in an hour and 120 in one whole day. Altogether more than 1,000 sets of gravity data were obtained. No attendance to the instrument was necessary. The ship's position and speed were determined as frequently as practicable with utmost care. Continuous records of echo sounding were also taken. The condition of the sea varied from calm to smooth-to-slight swells during the voyages. The vertical acceleration of the ship occasionally exceeded 50 gals.

The gravity data obtained are now being worked out by means of a NEAC 2203 computer. The Eötvös effects has been corrected for but the Browne correction need not be applied because the vibration string of the instrument kept the vertical direction with an accuracy already mentioned. The overall accuracy obtained is estimated to be 3–5 mgals. This is believed to be about sufficient for observations at open seas until difficulties in determining accurately the ship's position, speed, sea-bottom topographies and many other relevant factors can be overcome.

In the Bay of Tokyo which is about 30 km wide, there are gravimeter measurements along the coast lines on both sides so that gravity anomaly values along the ship's track in the bay can be approximately interpolated. The anomalies as measured by the present instrument and those interpolated agree reasonably well. The data obtained during the second voyage to Kagoshima are now being worked out and the results will be published shortly.

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